CONCERNING THE ALTERNATING-SIGN DISSIPATION OF ENERGY IN A FLUID WITH RELAXING VISCOUS STRESSES

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Total equations of motion of an incompressible fluid are investigated taking account of relaxation phenomena for viscous stresses and heat flux. In the K. I. Strakhovich class of solutions the fluid flows are considered that contain a strong hydrodynamic discontinuity. The conditions of motion are analyzed under which the dissipative function is negative.

Mass, momentum, and energy are transferred with a finite rate. This fact plays a great part in scientific problems associated with fast thermal and hydrodynamic processes in natural phenomena and in physical and power engineering equipment. The object of the present study is the effect of the finite velocity of the propagation of perturbations (relaxation of viscous stresses, relaxation of a heat flux) on the dissipation of mechanical energy in motion of an incompressible fluid. The objectives of the present work were: 1) to construct physically informative new analytical solutions for the total equations of motion of a viscous heat-conducting fluid; 2) to analyze the behavior of the dynamic and thermal parameters of an incompressible flow that contains a strong hydrodynamic discontinuity; 3) to reveal conditions under which the dissipative function becomes negative.

The plane two-dimensional nonstationary flow of an incompressible continuous medium is determined by the equations [1]:

$$\rho \frac{dv_i}{dt} = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ik}}{\partial x_k}; \quad \frac{\partial v_k}{\partial x_k} = 0; \quad i, k = 1, 2; \tag{1}$$

$$\rho c_p \frac{dT}{dt} = - \frac{\partial q_k}{\partial x_k} + \Phi; \quad \frac{d}{dt} = \frac{\partial}{\partial t} + v_k \frac{\partial}{\partial x_k}; \tag{2}$$

$$\Phi = \tau_{11} \frac{\partial v_1}{\partial x_1} + \tau_{22} \frac{\partial v_2}{\partial x_2} + \tau_{12} \left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} \right), \quad \rho, c_p - \text{const}.$$

To allow for the relaxation of viscous stresses, we use the Maxwell rheological equation of state of an viscoelastic fluid [2]

$$\tau_{ij} + \gamma \frac{D\tau_{ij}}{Dt} = 2\mu e_{ij}, \tag{3}$$

$$\frac{DR_{ij}}{Dt} = \frac{dR_{ij}}{dt} + m \left[R_{ik}\, \omega_{kj} - \omega_{ik}\, R_{kj} - l\, (R_{ik}\, e_{kj} + e_{ik}\, R_{kj}) \right]. \tag{4}$$

when $m = 0$, differentiation operator (4) is a substantive time derivative; when $m = 1$, $l = 0$ is a Jaumann convective derivative; when $m = 1$, $l = \pm 1$ are two Oldroid derivatives.

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The rheodynamic behavior of a nonlinear viscoplastic fluid is determined by Z. P. Shul'man's generalized model [3]

$$\tau_{ij} = 2\left[\frac{\tau_0^{1/n}}{A_1^{1/m}} + \mu^{1/m}\right]^n A_1^{n/m-1} e_{ij}, \quad A_1 = (2e_{ik}e_{ki})^{1/2}, \tag{5}$$

where τ_0 is the yield limit; n/m is the flow index. The classical model of a viscous Newtonian fluid has the form $\tau_{ij} = 2\mu e_{ij}$.

To describe heat transfer, we use the Fourier law $q_i = -\lambda \partial T/\partial x_i$, as well as the Maxwell-type relaxation model [4-6]

$$q_i + \gamma_1 \frac{dq_i}{dt} = -\lambda \frac{\partial T}{\partial x_i}, \quad i = 1, 2. \tag{6}$$

Certain problems of theoretical substantiation of the Navier–Stokes equations with allowance for the relaxation of viscous stresses and for heat-flux relaxation are also given in [6-8]. The existence of a negative dissipative function was noted earlier for turbulent flows in [9] and for an accelerating compressible gas flow in [8].

A line of strong discontinuity in incompressible fluid flow can have various physical origins. In particular, it is an effective model of a technological device in flowing through which the parameters of the fluid (density, viscosity, pressure, etc.) change sharply. The dynamic conditions of compatibility on the line of the strong discontinuity have the form [1]:

$$\{\rho (N - v_n)\} = 0, \quad \{\mathbf{p}_n + \rho \mathbf{v} (N - v_n)\} = 0, \tag{7}$$

$$\left\{\mathbf{p}_n \cdot \mathbf{v} + \rho (N - v_n)\left(\frac{\mathbf{v}^2}{2} + c_p T\right)\right\} = 0,$$

where $\mathbf{p}_n$ is the vector of surface stresses; v_n is the velocity component normal to the discontinuity line; N is the displacement velocity of the discontinuity line.

In [10] K. I. Strakhovich indicated (without giving a hydrodynamic interpretation) a class of exact solutions for the equations of isothermal motion of a Newtonian fluid. In the present work this approach is developed and generalized to the case of nonisothermal flow of rheologically complex fluids. Namely, on the basis of Eqs. (1)-(4), (6) in the class of flows

$$v_1 = -b \equiv \text{const}, \quad v_2 = \partial V(x, t)/\partial x, \quad p - p_0(t) = \rho\,\sigma_{11}(t),$$

$$\sigma_{11} + \sigma_{22} = 0, \quad \sigma_{ij} = \tau_{ij}/\rho, \quad F_1 \equiv 0, \quad F_2 = F \equiv \text{const},$$

$$\sigma_{12} = \frac{\partial V}{\partial t} - b\frac{\partial V}{\partial x} - xF + C(t), \quad t \geq 0,$$

$$T = T(x, t), \quad q_1 = q(x, t), \quad q_2 \equiv 0$$

we will consider a self-similar variant in which the desired functions depend on one argument $h = x + at$, $a \equiv$ const. When $m = 0.1$, $l = 0$, we have

$$v_2 \equiv \frac{dV}{dh} = (\sigma_{12} + hF)/(a - b), \quad \psi = \nu - \gamma(a - b)^2,$$

$$(\psi + m\gamma\sigma_{11}) \frac{d\sigma_{12}}{dh} = \sigma_{12}(a-b) - \nu F - m\gamma F\sigma_{11}, \quad p_0 \equiv \text{const}, \tag{8}$$

$$\left[\sigma_{11} + \gamma(a-b)\frac{d\sigma_{11}}{dh}\right](\psi + m\gamma\sigma_{11}) = m\gamma\sigma_{12}\left[\gamma F(a-b) - \sigma_{12}\right].$$

These equations are correct for both isothermal and nonisothermal flows and they do not contain constraints on the rheological model of viscosity.

Assume that a strong hydrodynamic discontinuity moves with a constant velocity: $x_j = -at$, $a < 0$. On one side of the discontinuity the fluid is immobile:

$$v_{1*} = 0, \quad v_{2*} = 0, \quad q_* = 0, \quad \overset{*}{\tau}_{ij} = 0, \quad \rho_*, \quad T_*, \quad \rho_* - \text{const};$$

on the other side, the flow is determined by formulas (8); dynamic compatibility conditions (7) are of the form:

$$A \equiv \rho(a-b) = \rho_* a < 0, \quad a \neq b, \quad p_0 - Ab = p_*, \quad \tau_{12j} = Av_{2j},$$

$$q_j + Ac_p(T_j - T_*) = bp_* + A(v_{2j}^2 + b^2)/2.$$

First, using Eqs. (8), we will consider the isothermal flow at ν, γ = const. Let $m = 1$ (Jaumann derivative); then for $\sigma_{11}(h)$, $\sigma_{12}(h)$ we obtain a dynamic system which has a singular point with the coordinates:

$$\overset{1}{c}_{12} = cw_0^2/(c^2+1), \quad \overset{1}{c}_{11} = -w_0^2/(c^2+1),$$
$$c_{12} = \sigma_{12}/(\gamma F)^2, \quad c_{11} = \sigma_{11}/(\gamma F)^2, \quad w_0^2 = w^2/(\gamma F)^2, \quad c = (a-b)/(\gamma F). \tag{9}$$

Analysis has shown that the following variants are possible: 1) $c^2 + 1 < w_0^2$, singular point (9) is a saddle; 2) $w_0^2 < (c^2+1) < 2w_0^2$, $c^2 < 1$, and then for $c > 0$ there is an unstable node and for $c < 0$ a stable node; 3) for $9-\sqrt{80} < c^2 < 1$ point (9) is a focus, which is unstable for $c > 0$ and stable for $c < 0$. In the given class of solutions the singular point "center" is absent.

Thus, the stability or instability of singular point (9) depends on the sign of the complex c that carries information on the two-dimensionality of the flow, namely, on the orientation of the force F and on the fluid mass flow through the discontinuity. We also note that in formula (8) for v_2 the first term characterizes the effect of viscous friction forces, and the sign of the number c determines the sign of the second term, which describes the effect of the mass force on the transverse velocity.

In what follows we will consider two variants: 1) a flow downstream of the discontinuity in a semi-infinite region, $x \in (-\infty, x_j)$; 2) a flow in the j-region between two strong discontinuities, $x \in [x_s, x_j]$, $x_s = h_s - at$, $h_s < 0$. Downstream of the second discontinuity we assume a uniform flow:

$$v_1 = -b_{**}, \quad v_2 = v_{**}, \quad p = p_{**}, \quad T = T_{**}, \quad \overset{**}{\tau}_{ij} \equiv 0.$$

The conditions of dynamic compatibility on the second discontinuity are

$$x = x_s: \; A = \rho(a-b) = \rho_{**}(a - b_{**}), \quad p_{**} - p_* = Ab_{**},$$

$$\tau_{12,s} = A(v_{2s} - v_{**}),$$

$$Ac_p(T_s - T_{**}) = (b - b_{**})\left[p_* + \frac{A}{2}(b + b_{**})\right] + (\tau_{12,s})^2/(2A).$$

This approach allows us to assume that the heat capacity is piecewise constant; here, to simplify the representation of the formulas we assume that $c_p = c_p^* = c_p^{**}$. The fluid mass flow through the j-region is characterized by the number $A < 0$.

We note that the flow between two strong discontinuities was applied in gas dynamics [11] as a model of a piston with longitudinal holes. The nonlinear dynamic and thermal properties of the vorticity curl in a viscoelastic fluid flow containing a strong hydrodynamic discontinuity was investigated in [12-14].

If there is isothermal flow in the absence of mass force ($F = 0$), then at $m = 0$ for the vorticity we have $2\omega = \sigma_{12}/\psi$. This means that the vorticity curl is directly proportional to the viscous tangential stress if the fluid is either Newtonian or viscoelastic with the operator of the substantive derivative in the rheological equation of state. A linear relationship between ω and τ_{12} for certain isothermal and nonisothermal flows of Newtonian and viscoelastic fluids was noted earlier in [12-14]. If relaxation of viscous stresses is absent ($\gamma = 0$) and the fluid is nonlinearly viscoplastic (5), then in the class of motions (8) $\tau_{12} = \tau_{12}(\omega)$ is the fractional power function

$$\tau_{12} = (2\omega)^{n/m} \left[\frac{\tau_0^{1/n}}{(2\omega)^{1/m}} + \mu^{1/m} \right]^n .$$

Next, we will consider a nonisothermal flow and assume that

$$\nu = \nu_0 f , \quad \gamma = \gamma_0 f , \quad \lambda = \lambda_0 f , \quad \gamma_1 = \gamma_{10} f , \quad f = f(T) > 0 , \quad c_p \equiv \text{const} ,$$

i.e., the shear-wave velocity and the speed of heat propagation are constant. Then, the solution downstream of the discontinuity $h_j = 0$ has the form:

$$v_1 = -b < 0 , \quad v_2 = \sigma_{12}/(a - b) , \quad p - p_0 = \rho\sigma_{11} , \quad m = 0 ,$$

$$dh = \frac{\psi_0 f(T)}{a - b} ds , \quad \psi_0 = \nu_0 - \gamma_0 (a - b)^2 , \quad \sigma = \frac{\sigma_{12}}{\sigma_{12,j}} , \quad \sigma \in (0, 1] ,$$

$$\sigma_{11} = \sigma_{11,j} \exp \left[-\frac{h}{\gamma (a - b)} \right] , \quad q = \frac{\rho \sigma_{12}^2}{2 (a - b)} - A c_p (T - T_\infty) , \quad \Pr = \frac{c_p \mu}{\lambda} .$$

The temperature is represented in the dimensionless form $\theta = (T - T_\infty)/(T_j - T_\infty)$; the Newtonian fluid, $\gamma \equiv 0$, $\gamma_1 \equiv 0$,

$$\theta = \sigma^\alpha - \frac{(\sigma^2 - \sigma^\alpha) \Pr \mathrm{Ec}}{2 (2 - \alpha)} , \quad \alpha = \Pr , \quad \Phi > 0 :$$

the viscoelastic fluid displaying Fourier heat conduction, $\gamma_1 \equiv 0$,

$$\theta = \sigma^\alpha - \frac{(\sigma^2 - \sigma^\alpha) \Pr \mathrm{Ec} (1 - \mathrm{M}^2)}{2 (2 - \alpha)} , \quad \alpha = (1 - \mathrm{M}^2) \Pr ; \tag{10}$$

the viscoelastic fluid with a relaxing heat flux

$$\theta = \sigma^\alpha - \frac{(\sigma^2 - \sigma^\alpha) \mathrm{Ec} [\gamma_1 \gamma^{-1} + (\mathrm{M}^{-2} - 1) 2^{-1}]}{[\mathrm{M}^{-2} (\Pr)^{-1} - \gamma_1 \gamma^{-1}] (2 - \alpha)} , \quad \alpha = \frac{\gamma \mathrm{M}_1^2 (1 - \mathrm{M}^2)}{\gamma_1 \mathrm{M}^2 (1 - \mathrm{M}_1^2)} . \tag{11}$$

Here, the Eckert number contains the fluid slip velocity on the internal side of the discontinuity, $\mathrm{Ec} = v_{2j}^2/[c_p(T_j - T_\infty)]$.

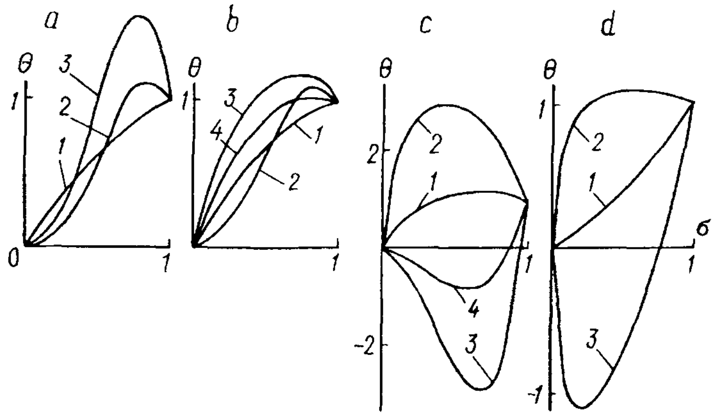

Fig. 1. Relationship between temperature and viscous tangential stress: a) Newtonian fluid; 1) Pr = 1, Ec = 1; 2) 5 and 3; 3) 5 and 5; b) viscoelastic fluid with Fourier heat conduction, $\Phi > 0$; 1) Pr = 1, Ec = 1, M = 0.75; 2) 5, 3, and 0.5; 3) 5, 5, and 0.9; 4) 5, 3, and 0.9; c) viscoelastic fluid with thermal relaxation, $\Phi > 0$, $\gamma/\gamma_1 = 0.3$; 1) Pr = 1, Ec = 1, M = 0.5, $M_1 = 0.6$; 2)5, 2, 0.5, and 0.6; 3) 5, 3, 03, and 0.99; 4) 5, 1, 0.9, and 0.99; d) viscoelastic fluid with thermal relaxation, $\Phi < 0$, $\gamma/\gamma_1 = 0.3$, M = 1.1, $M_1 = 1.2$; 1) Pr = 1, Ec = 1; 2) 5 and 1; 3) 5 and 3.

In the semi-infinite region the boundedness condition for solving Eqs. (10) and (11) has the form: $\alpha > 0$. This means that the flow of a viscoelastic fluid having Fourier heat conduction occurs in a subsonic regime, $M^2 < 1$; if there is also thermal relaxation, then we should have $M^2 < 1$, $M_1^2 < 1$ or $M^2 > 1$, $M_1^2 > 1$.

From the formula for the dissipative function $\Phi = \rho\sigma_{12}^2/\psi$ we conclude that when $M^2 > 1$, the dissipative heat generation can be negative in the supersonic viscoelastic flow. Consequently, when $M^2 > 1$, $M_1^2 > 1$, we will have $\Phi < 0$ for $\gamma > 0$ and $\gamma_1 > 0$ in the semi-infinite region downstream of the discontinuity. In order to obtain $\Phi < 0$ in a fluid with Fourier heat conduction, we must set up still another strong discontinuity $h = h_s < 0$, $v_{**} = 0$, $h \in \{h_s, 0\}$. The solution (10), in which $T_\infty = T_0$ and $\alpha < 0$, is physically meaningful if both discontinuities are the jumps of heating, $T_j > T_*$, $T_s < T_{**}$ and if at a small but the finite value of $\sigma_s \in (0, 1)$ the following estimates are satisfied:

$$\rho_{**} < \rho < \rho_* , \quad p_* > p_{**} > 0 , \quad T_{**} > T_* > T_0 , \quad b_{**} > b > 0 ,$$

$$2p_* > a(a - b)(\rho_* - \rho) , \quad 2(2 - \alpha) + \alpha \, \mathrm{Ec} < 0 ,$$

where

$$c_p (T_0 - T_*) = p_* b A^{-1} + \frac{b^2}{2} , \quad T_s < T_j .$$

We will give some calculation results typical for this problem. The dependences $\theta = \theta(\sigma)$ are presented in Figs. 1a-1d. From these figures we conclude that the effect of the parameter Ec (in particular, of the slip velocity) on temperature is qualitatively the same both in the Newtonian and viscoelastic subsonic variants (Figs. 1a and 1b). If thermal relaxation is taken into account, then the temperature fields in the subsonic variants (Fig. 1c) differ considerably from one another, depending on the interval in which the number α is located: either $\alpha \in (0, 1)$ (lines 1, 2, $\alpha \cong 0.5$) or $\alpha > 1$ (lines 3, 4, $\alpha \cong 3.5$). In the supersonic regime with negative dissipation (Fig. 1d) a strong, both quantitative and qualitative, effect of Ec on $\theta(\sigma)$ is observed.

Let us consider the flow in the j-region between two discontinuities with allowance for the mass force $F \neq 0$. We begin with the particular ("sonic") variant in which $\psi = 0$, i.e., $M^2 = 1$. From formulas (8) at $m = 0$ we find

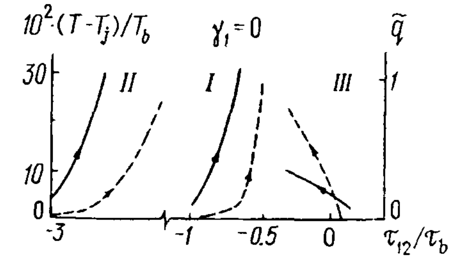

Fig. 2. Properties of heat field for viscoelastic fluid with Fourier heat conduction.

$$\sigma_{12} = F\mu(T) A^{-1}, \quad \Phi = \frac{F^2\mu}{(a-b)^2}\left[1 + \frac{\dot{\mu}(T)}{A}\frac{dT}{dh}\right]. \tag{12}$$

We can recognize conditions under which $\Phi < 0$ in the vicinity of the discontinuity $h_j = 0$. Here the effect of negative dissipation is due to the temperature dependence of viscosity. For the sake of definiteness we assume that $\dot{\mu}(T) < 0$, which is true for many incompressible fluids. Analysis of solution (12) makes it possible to establish that: 1) the discontinuity $h_j = 0$ must be a jump in heating, $T_j > T_*$, $\rho < \rho_*$, and the discontinuity $h = h_s < 0$, a jump in cooling, $T_s > T_{**}$, $\rho < \rho_{**}$; 2) the "thermal" Mach number is smaller than unity, $M_1^2 < 1$; 3) it is necessary to satisfy the following joint inequalities that allow one to estimate p_*, F^2 and the temperature jump:

$$F^2\gamma_1\gamma < (M_1^2 - 1)\frac{\lambda}{\dot{\mu}} < F^2\gamma_1\gamma - (b_{**}p_*/A) - b_{**}^2;$$

$$-Ab_{**} < p_* < -\frac{A}{2}(b + b_{**}), \quad b > b_{**} = a\left(1 - \frac{\rho_*}{\rho_{**}}\right); \tag{13}$$

$$2c_p(T_j - T_*) > F^2\gamma^2\left(1 + \frac{2\gamma_1}{\gamma}\right) - b^2, \quad v_{**} = \rho F h_s/A.$$

The acquisition of these estimates terminates the study of the "sonic" variant. Note that at $F \neq 0$ the solutions presented here have a local character along the coordinate y and are exact on the line $y = 0$.

The directions of the force F and of the fluid mass flow through the j-region are orthogonal to each other. Let us discuss the effect of this two-dimension factor on the structure of the solution.

We assume that the temperatures of the fluid in the regions (*), (j), and (**) are constant and different; $q_j = q_s = 0$. When $m = 1$, the analysis of singular points (9) in the region between two discontinuities is valid. The solution has a physical meaning if the fluid is heated at the first discontinuity, $T > T_*$, $\rho < \rho_*$, and cooled at the second, $T_{**} < T$, $\rho < \rho_{**}$ and if inequalities (13) are satisfied.

The Maxwell–Oldroid self-similar fluid flow ($m = 1$, $l = 1$) in the j-region between two discontinuities for c_p, μ, λ, γ, γ_1 = const admits an analytical description:

$$p - \tau_{11} = p_0 \equiv \text{const}, \quad v_1 = -b, \quad v_2 = (\sigma_{12} + hF)/(a-b),$$

$$\sigma_{11} = s_{11}/E, \quad E = \exp z, \quad z = h/[\gamma(a-b)], \quad h \in [h_s, 0].$$

In an isobaric case ($s_{11} = 0$), when the pressures of the fluid in the regions (*), (j), and (**) are constant and different, we have:

$$\tau_{12}/\tau_{12,b} = 1 + (v_0 - 1)(1 + E_1), \quad \tau_{12,b} = \mu F/(a-b),$$

$$T_b = \mu\gamma^2F^2/\lambda, \quad q_b = \gamma(a-b)\Phi_b, \quad \Phi_b = \mu F^2/(a-b)^2,$$

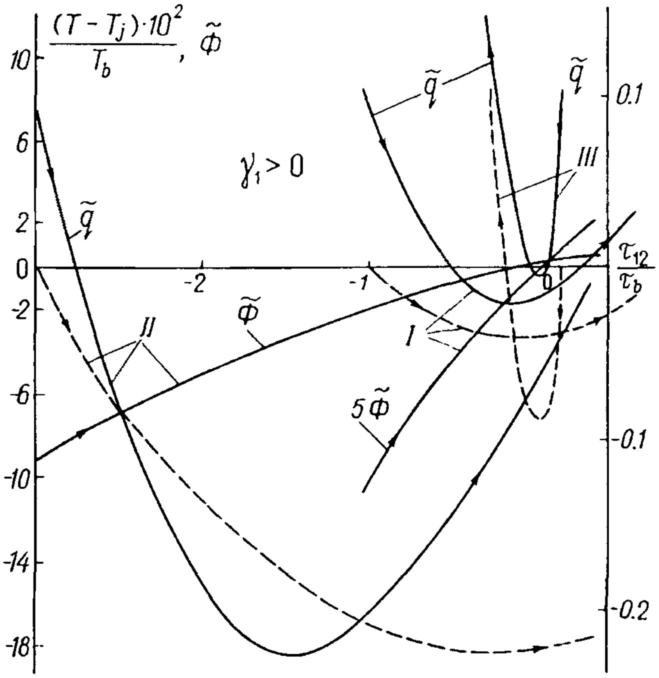

Fig. 3. Properties of heat field for fluid with viscous and thermal relaxation.

$$\frac{(T - T_j)}{T_b} = M_2^{-1}\left[\frac{k_j}{D}(1 - E^{-D}) - \frac{\psi_1 E_1}{D_1 M_2 d_1} + \frac{\psi_2 E_2}{D_2 M_2 d_2} - \frac{z}{M_2 D}\right] + \frac{\gamma_1}{\gamma M_2}\left[\frac{\psi_1 E_1}{d_1} - \frac{\psi_2 E_2}{d_2} + z\right];$$

$$\frac{q}{q_b} = k_j E^{-D} - \frac{1}{M_2}\left[\frac{1}{D} + \frac{\psi_1}{D_1}(1 + E_1) - \frac{\psi_2}{D_2}(1 + E_2)\right];$$

$$\psi_1 = \frac{(\nu_0 - 1)(M^2 - 2)}{(M^2 - 1)}, \quad \psi_2 = \frac{(\nu_0 - 1)^2}{(M^2 - 1)}, \quad \nu_0 = \frac{\nu_{2j}(a - b)^2}{\nu F},$$

$$k_j = \frac{q_j}{q_b} - \frac{1}{M_2}\left(\frac{\psi_2}{D_2} - \frac{\psi_1}{D_1} - \frac{1}{D}\right), \quad M_2 = M^2 \Pr \frac{\gamma_1}{\gamma} - 1, \quad D = \frac{M^2 \Pr}{M_2},$$

$$d_1 = M^2/(1 - M^2), \quad d_2 = 2d_1, \quad D_i = D + d_i, \quad E_i = E^{d_i} - 1, \quad i = 1, 2;$$

$$\tilde{\Phi} \equiv \frac{\Phi}{\Phi_b} = 1 + \psi_1(1 + E_1) - \psi_2(1 + E_2). \tag{14}$$

Here ν_0 is the Froude number. From Eq. (14) it follows that the dissipative function depends nonlinearly on the viscoelastic Mach number. The effect of negative dissipative heat generation is observed in both supersonic and subsonic flow regimes. If $M^2 > 1$, then $\Phi_j < 0$ in two cases: 1) the Froude number is negative, $\nu_0 < 0$, i.e., the directions of the mass force and slip velocity are opposite; 2) the Froude number is positive and such that $(\nu_0/M^2) > 1$. If $M^2 < 1$, then $\Phi_j < 0$ when $0 < \nu_0 M^{-2} < 1$. This means that if the directions of the mass force and the slip velocity are the same, then the appearance of negative dissipation in a subsonic or supersonic flow depends on the magnitude of the fraction $\nu_0/M^2 = \nu_{2j}/(\gamma F)$.

We also indicate that the parabolic-type (of the second power) dependence of the negative dissipative function on the Froude number has a maximum in a supersonic flow ($1 < M^2 < 2$) at the point $z = z^1$ for which

$$\nu_0^1 = 1 + \frac{(M^2 - 2)}{2(1 + E_1(z^1))} < 0.$$

In the remaining cases the function $\Phi = \Phi(v_0) < 0$ is monotonic. In the given solution the viscous tangential stress is linearly related to the Froude number; therefore, the function $\Phi(\tau_{12})$ is also parabolic. Figures 2 and 3 illustrate the dimensionless dependences of the temperature (dashed lines), heat flux $\tilde{q} = q/(-q_b)$, and dissipative function $\tilde{\Phi}$ on viscous stress for viscoelastic fluids with Fourier thermal conductivity (Fig. 2) and with thermal relaxation (Fig. 3), $\gamma_1/\gamma = 0.3$. It was assumed in the calculations that $\Pr = 5$, $q_j/(-q_b) = 0.1$. Shown here are three examples: I. $M^2 = 3$; $v_0 = -1$; II. $M^2 = 3$; $v_0 = -3$; III. $M^2 = 1/4$; $v_0 = 0.1$. Two cases represent a supersonic flow and the third case gives a subsonic flow, with $\tilde{\Phi}_j < 0$ in each variant. The arrow along the curves indicates the direction of evolution of the corresponding function from its initial value on the right discontinuity $h_j = 0$ toward the left boundary $h = h_s$. In example III the dissipative function increases monotonically in the interval $[-0.02; 0.27]$ (not shown in Fig. 3). When $\gamma_1 = 0$, $\gamma > 0$, the subsonic and supersonic variants are described by monotonic dependences and differ from each other by the directions of convexity and by the angles (obtuse and acute) of inclination of the curves to the axis of stresses.

Heat-flux relaxation changes the temperature field substantially (Fig. 3): T and q become nonmonotonous functions of the tangential stress. An increase in $|v_0|$ changes the quantitative characteristics of hydrodynamic and thermal fields: this effect is especially strong when $\gamma_1 > 0$.

If $s_{11} \neq 0$ and the pressure is variable in the j-region, a solution can be obtained in quadratures and in elementary functions for particular values of the Mach number (for example, $M^2 = 0.5$; 2). For the given flow the condition that $\Phi_j < 0$ coincides with an isobaric case.

Let us summarize the results. We presented model theoretical concepts that make it possible to judge the possibility of the existence of alternating-sign dissipative heat generation in an incompressible fluid flow. The investigation is based on an analysis of the class of Eq. (8)-type motions in the presence of a strong hydrodynamic discontinuity. Two models of heat transfer are considered: Fourier heat conduction and Maxwell-relaxing heat flux. The relaxation of viscous stresses is a necessary condition of the negative dissipative function. If a mass force is absent, the anomaly $\Phi < 0$ is possible when $M^2 > 1$. A mass force whose direction is orthogonal to that of the discontinuity motion has a substantial effect on energy dissipation in a Maxwell–Oldroid fluid. The quantity $v_{2j}/(\gamma F)$, which characterizes the mutual orientation of the mass force and the fluid velocity vectors on the discontinuity, is a quantitative criterion here.

NOTATION

$x_1 = x$, $x_2 = y$, Cartesian rectangular coordinates; t, time; v_1, v_2, velocity vector components; p, pressure; ρ, density; T, temperature; q_1, q_2, components of specific heat flux vector; τ_{ij}, components of stress tensor deviator; e_{ij}, components of deformation rate tensor; $2\omega_{ij} = (\partial v_i/\partial x_j) - (\partial v_j/\partial x_i)$; F, mass force; c_p, specific heat; γ, relaxation time of viscous stresses; γ_1, relaxation time of heat flux; μ, dynamic viscosity coefficient; $\nu = \mu/\rho$; λ, thermal conductivity coefficient; ω, vorticity; M, viscoelastic Mach number, $M^2 = (a - b)^2/w^2$; M_1, "thermal" Mach number, $M_1^2 = (a - b)^2/w_1^2$; $w^2 = \nu/\gamma$; $w_1^2 = \lambda/(\rho c_p \gamma_1)$; $\{f\} = f_1 - f_2$, jump of the function in passing through discontinuity; $N = -a$, displacement rate of discontinuity; Φ, dissipative function. Superscripts and subscripts: a point above the sign of the function denotes differentiation with respect to its argument, the repeating index denotes summation; *, parameters of fluid before discontinuity; **, parameters of fluid behind second discontinuity; j and s, values of functions on the right and left boundaries of the region between two discontinuities; ∞, parameters of the fluid at an infinite distance from the discontinuity; b, scales of quantities.

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